Quantum phase transition from an antiferromagnet to a spin liquid in a metal

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We study quantum phase transitions from easy-plane antiferromagnetic metals to paramagnetic metals in Kondo-Heisenberg lattice systems. If the paramagnetic metal is a fractionalized Fermi liquid then the universal critical properties of the phase transition are unaffected for a weak Kondo coupling even when the Fermi surface intersects the magnetic zone boundary. This is in striking contrast to the conventional theory of phase transitions between paramagnetic and antiferromagnetic metals where any Kondo coupling is strongly relevant, and leads to a Landau-damped "Hertz-Millis" theory. The electron quasiparticle remains well defined in the quantum critical regime and the critical spin fluctuations only contribute subleading corrections to the various properties of conduction electrons.

I. INTRODUCTION

The earliest and the most influential approach to study magnetic quantum phase transitions in metals [developed by Hertz, Moriya, Millis[1] and others (for additional references see, e.g., Ref. [2])] focuses on the fluctuations of the natural magnetic order parameter. The metallic environment results in damping of these magnetic fluctuations. The resulting theory is however weakly coupled in physical dimension $d \geq 2$—consequently it allows only for weak deviations from Fermi-liquid physics even right at the magnetic quantum critical point. In particular, the spin-fluctuation spectrum at finite temperature T and a frequency ω does not show scaling as a function of ω/T.

Experimentally the best-studied magnetic quantum phase transitions are in heavy-fermion metals[2] which have a lattice of local moments coupled via Kondo exchange to a partially filled band of conduction electrons. Remarkably in many cases, striking breakdown of Fermi-liquid physics is observed near these quantum critical points which are not understood even qualitatively. For instance, the spin-fluctuation spectrum in $CeCu_{6-x}Au_x$ at quantum criticality is scale invariant and shows ω/T scaling.[3–5] A number of transport and thermodynamic anomalies have also been seen in this and other materials.[6–8] In the heavy-fermion context, the Hertz-Moriya-Millis (HMM) approach treats the magnetism as a spin-density wave instability of the heavy Fermi liquid and essentially assumes that the Kondo process responsible for the formation of the Fermi-liquid state continues across the magnetic transition. An alternate approach specific to heavy electron metals suggests that the transition to magnetism is accompanied by the breakdown of the Kondo effect.[9–12] Empirically this will manifest itself as a reconstruction of the electronic Fermi surface associated with the loss of participation of the local moments in the Fermi sea. Evidence in favor of such a Fermi-surface reconstruction accompanying the magnetic ordering transition has been presented in $YbRh_2Si_2$ (Ref. [13]) and in $CeRhIn_5$.[14] Very recently studies of $YbRh_2Si_2$ with Co or Ir substituted at the Rh site have suggested that the Fermi-surface reconstruction may be separated from the magnetic transition through chemical pressure.[15] Most interestingly with Ir substitution, the anti-ferromagnetic (AFM) metal appears to be separated from the paramagnetic large Fermi-surface metal through an intermediate phase which has neither magnetism nor Kondo screening. The nature of this intermediate phase and the associated quantum phase transitions remains to be elucidated.

On the theoretical side, much attention has focused on the possible quantum phases and phase transitions of Kondo lattices driven by competition between intermoment exchange and the Kondo effect. Two common phases are of course a magnetically ordered metal with a "small" conduction-electron Fermi surface, and a paramagnetic heavy Fermi liquid with a "large" Fermi surface. An intriguing third possibility is that the local moments form a quantum spin-liquid state which then remains decoupled from the conduction electrons.[11,12,16] The resulting paramagnetic metal will have a small Fermi surface and hence violate the usual Luttinger theorem.[11] Such phases were dubbed "fractionalized Fermi liquids" and denoted FL*. Quantum phase transitions in Kondo lattice systems are much less understood. The possibility of a single direct transition where the magnetic ordering is accompanied by the destruction of the Kondo effect (and the associated Fermi-surface reconstruction) has been discussed but without satisfactory resolution. Progress has been possible in analyzing phase transitions associated with the breakdown of the Kondo effect without worrying about the magnetic ordering. A concrete theory of such a "Kondo breakdown" transition from the usual heavy Fermi liquid to a "small Fermi surface" metal was proposed in Ref. [17] and further developed in Ref. [18].

In this paper, we explore instead the complementary problem of the quantum phase transition associated with the destruction of magnetism without worrying about the onset of Kondo screening. Specifically we study the phase transition between a magnetic metal and a particular fractionalized Fermi liquid where the local moments form a quantum spin liquid. We focus on the interesting case where the magnetic ordering wave vector can connect distinct points of the conduction-electron Fermi surface. Remarkably we find that the magnetic phase transition remains strongly coupled, and satisfies nontrivial scaling (such as in the spin-fluctuation spectrum as a function of ω/T). This is in striking contrast to the usual HMM spin-density wave transition. We also study the effects of these critical spin fluctuations on the properties

of the electronic quasiparticles of the quantum critical metal.

Our considerations will be based on Kondo-Heisenberg models with the general structure,

$$H = H_c + H_K + H_{int}, \tag{1}$$

$$H_c = \sum_k \epsilon_k c_k^\dagger c_k, \tag{2}$$

$$H_K = \frac{J_K}{2} \sum_r \vec{S}_r \cdot c_r^\dagger \vec{\sigma} c_r, \tag{3}$$

$$H_{int} = \sum_{rr'} J_a S_r^a S_{r'}^a + \cdots. \tag{4}$$

Here $c_{k\alpha}$ destroys a conduction electron of momentum k and spin α and $\vec{S}_r$ are spin-1/2 operators describing the localized moments. In the last term, we will allow for other short-ranged interactions between the local moments that enable tuning the local-moment system through magnetic phase transitions.

Generally the magnetic phase transitions of the kind we wish to study are conveniently accessed theoretically by first ignoring any coupling between conduction electrons and the local moments. Then the local-moment system is insulating and much is known about the phases and phase transitions of such quantum magnets. The general question we are concerned with is the effect of a small coupling of this insulating quantum magnet to the metallic electrons. The best understood example of such a quantum phase transition in an insulating magnet is that between a Neel state and a gapped quantum spin liquid. In the simplest such spin-liquid state, the excitations consist of spin-1/2 bosonic spinons. In addition, there are gapped Z_2 vortices (visons) that act as sources of π flux for the spinons. Second-order quantum phase transitions to such states from magnetically ordered states are known to be possible in two cases—first for magnets with noncollinear magnetization patterns and second for magnets with easy-plane anisotropy with either collinear or noncollinear magnetic order. For concreteness, we will focus on easy-plane quantum antiferromagnets with collinear order in this paper.

In the presence of weak coupling to the metallic electrons Ref. 11 argued that the quantum spin-liquid state and its topological order are stable. The result is a "fractionalized Fermi liquid" phase which has a c-electron Fermi surface coexisting with gapped fractionalized spinon excitations. The transition to the Neel state has not been previously described and will be studied in this paper. We first argue that a weak coupling to the c electrons actually stabilizes the Neel state relative to the spin-liquid state. Thus the phase boundary curves as shown in Fig. 1.

In the ordered state, the c electrons near the points connected by the ordering wave vector will be strongly affected and the Fermi surface will change shape compared to the paramagnet. However, we will argue that right at the magnetic quantum critical point, the damping of the spin fluctuations due to the metallic quasiparticles is *irrelevant*. Thus, the c electrons dynamically decouple from the critical spin
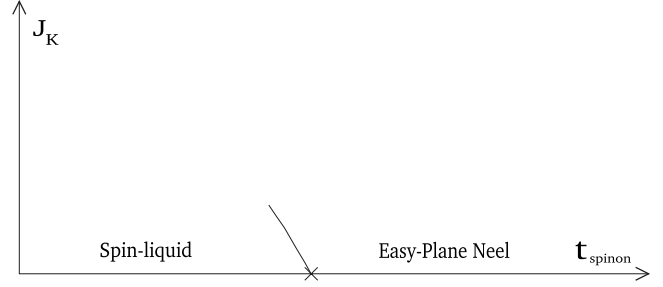

FIG. 1. Phase diagram in the presence of Kondo coupling. As shown in the text, weak Kondo coupling stabilizes the ordered Neel phase relative to the spin liquid.

fluctuations even though they are strongly coupled in the ordered state. This therefore provides an example of a strongly coupled magnetic quantum critical point in a metallic environment. We study the effect of the critical spin fluctuations on the c electrons. Near "hot spots" on the Fermi surface (points that are connected to each other by the ordering wave vector), there is anomalous scattering leading to electron lifetimes that vanish as $T^{\bar{\eta}}$ with $\bar{\eta} > 1$. Thus the quasiparticle picture of the c excitations is preserved at this quantum critical point. We describe the corrections to Fermi-liquid results in transport and other quantities that result from the anomalous scattering at these hot spots.

II. NEEL ORDER—QUANTUM SPIN-LIQUID PHASE TRANSITION IN EASY-PLANE ANTIFERROMAGNETS

Consider easy-plane antiferromagnets with spin-1/2 per unit cell with collinear order at a wave vector $\vec{Q}$ on square lattice. The spin ordering pattern is characterized in terms of an *XY* order parameter ψ,

$$S^-(\mathbf{r}_i) \sim e^{i\mathbf{Q} \cdot \mathbf{r}_i} \psi. \tag{5}$$

The *XY* ordering allows for vortex topological defects characterized by an integer winding number that are pointlike in two space dimensions. Now consider disordering the *XY* order to move into a quantum paramagnet by proliferating these vortices. When single vortices (i.e., those with 2π winding) proliferate, the result is a quantum paramagnet with broken translational symmetry (a valence-bond solid). Quantum spin-liquid states result if the *XY* order is killed by proliferating *paired* vortices (i.e., those with 4π winding). The resulting spin-liquid state is described as a gapped topologically ordered Z_2 spin liquid with bosonic spin-1/2 spinons, and a gapped Z_2 vortex (vison) that may be understood as an unpaired vortex. A lattice model exhibiting this physics was constructed in reference.[19]

As shown in Ref. 19, the quantum phase transition between this spin liquid and the ordered state may be understood as a condensation of the bosonic spinons. In terms of the spinon fields (denoted as $b^\dagger$), it is in the universality class of the 2+1 dimensional classical *XY* model. However, the physical spin order parameter is obtained as a square of the spinon field,
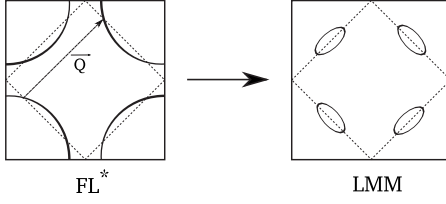
FIG. 2. Fermi Surface in the two phases FL* and LMM (local moment metal).

$$\psi = b^2. \tag{6}$$

Consequently, the order-parameter correlations decay with a large anomalous exponent,

$$\langle \psi^*(\mathbf{k}, \omega) \psi(\mathbf{k}, \omega) \rangle \sim \frac{1}{[k^2 - (\omega + i\epsilon)^2]^{1 - \bar{\eta}/2}} \tag{7}$$

with $\bar{\eta} = 1.45$ (see, for example, Ref. 20). We would refer to this phase transition as lying in the XY^* universality class in this paper since it is distinct from the usual XY phase transition where the physical (planar) spin could be identified with the order parameter.

III. COUPLING TO METALLIC ELECTRONS

A. Weak-coupling phase diagram

Now consider turning on a weak but nonzero interaction of the form in Eq. (3) between the insulating easy-plane magnet described above and a sea of noninteracting conduction electrons. We will focus on the effect of this interaction on the slow hydrodynamic variables of the insulating quantum magnet. These are simply the antiferromagnetic order parameter ψ, the conserved uniform spin density $\vec{L}$, and the conserved energy density ϵ. The order-parameter coupling is the most important and takes the form

$$S = \int_{\mathbf{q}, \mathbf{k}_1, \mathbf{k}_2, \tau} J_K \psi^+(\mathbf{q}) \cdot c_{\mathbf{k}_1}^\dagger \sigma^- c_{\mathbf{k}_2} \delta(\mathbf{q} + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{Q}) + \text{c.c.}, \tag{8}$$

where $\mathbf{Q}$ is the ordering wave vector. We will assume that $\mathbf{Q}$ can connect two distinct points on the conduction-electron Fermi surface (Fig. 2). The electrons near such Fermi-surface points scatter strongly off the fluctuations of the magnetic order parameter. In the ordered phase, this will open up a gap at such points of the conduction-electron Fermi surface. The size of the gap Δ_{el} will be determined by the strength of order parameter $\psi_0 = |\psi|$. Within mean-field theory, $\Delta_{el} \sim J_K \psi_0$. In the terminology of Ref. 17, this is a "local-moment magnetic metal." (LMM).

In the disordered spin-liquid phase, the order-parameter fluctuations are gapped. Consequently, they can be integrated out in favor of a short-ranged spin-fluctuation-mediated electron-electron interaction. In the absence of any special nesting condition on the Fermi surface (which we assume), this does not significantly affect the low-energy properties of the c electrons. Further, the volume of the Fermi surface is determined by the density of c electrons alone without count-

ing the local moments. As argued in Ref. 11, the fractionalization and topological order of the local-moment system survives the coupling to the metal. This metallic phase is a fractionalized Fermi liquid (FL*) which has spinon and associated topological excitations coexisting with a small Fermi surface of c electrons.

One important issue is the relative stability of the spin-liquid phase compared to the Neel phase in the metallic environment. In other words, does the coupling to the conduction electrons enhance or decrease the parameter regime in which the spin liquid exists? We argue now that at least at weak coupling, the Neel state gains in stability relative to the spin liquid. Physically this may be traced to the damping of the spin fluctuations due to particle-hole pairs of the Fermi liquid. In the paramagnetic phase, if we integrate out the conduction electrons, the (imaginary time) action for the Neel order parameter acquires the familiar Landau damping term $\int_{\vec{q}, \omega} |\omega| |\psi|^2$. To assess the effect of this damping on the phase diagram, consider the following simple model for the spinons b on a $2+1$ dimensional space-time cubic lattice with the action,

$$S = S_0 + S_1 = \sum_{\langle (\mathbf{x}, \tau), (\mathbf{x}', \tau') \rangle} -t[b^\dagger(\mathbf{x}, \tau) b(\mathbf{x}', \tau') + \text{H.c.}]$$
$$+ g \sum_{\mathbf{x}, \tau, \tau'} K(\tau - \tau') b^2(\mathbf{x}, \tau) b^{\dagger 2}(\mathbf{x}, \tau') \tag{9}$$

subject to the constraint $b^\dagger(\mathbf{x}, \tau) b(\mathbf{x}, \tau) = 1$. Here $K(\tau) = \frac{1}{\tau^2}$ is the imaginary time Fourier transform $|\omega|$. The constant $g \approx J_K^2 / v_F Q$, where v_F is the Fermi velocity. Note that the coupling term breaks the rotational invariance in $\mathbf{x}$-τ space. As t increases, at certain critical value the b field orders. Denoting this critical value in the absence and presence of g by t_{0c} and t_c, respectively, we find that $t_c = t_{0c} - \alpha g$, where $\alpha = O(1)$ is a positive constant. The details are given in the Appendix A.

B. Critical properties

We now turn to the critical properties of the transition in the presence of a weak coupling to the conduction electrons. The electrons will couple to the slowly varying component of the order-parameter field. The lowest-order addition to the Hamiltonian consistent with the symmetries of the lattice and spin rotation is

$$\Delta H_1 = J_K \int_r c^\dagger(r) \sigma^+ c(r) \cdot S^-(r) + \text{H.c.}, \tag{10}$$

where "g" is a coupling constant. Another potentially important term involves coupling between energy densities of the electron and local moments. To leading order, this takes the form

$$\Delta H_2 = \lambda \int_r [c^\dagger(r) c(r)] \cdot [b^\dagger(r) b(r)] \tag{11}$$

with λ as another coupling constant. Let us now integrate out the conduction electrons to obtain an effective action for the spinon fields. As already described above, the main effect of ΔH_1 is to generate the usual Landau damping term in the

action $S_1 = \int_{k,\omega} g|\omega||\psi(k,\omega)|^2$. From the point of view of renormalization group (RG), one crucial observation concerning S_1 is that the kernel $K(\omega)$ is long ranged in the time domain $[K(\tau) = \frac{1}{\tau^2}]$ and thus couples ψ's at two separate points in space time. This implies that at the tree level, one could directly use the scaling dimensions for individual ψ operator's even though $|\psi(k,\omega)|^2$ is a composite operator. Using the RG transformation appropriate for the critical fixed point in the absence of the conduction electrons, $\omega \rightarrow s\omega, k \rightarrow sk, \psi \rightarrow s^{(\bar{\eta}-5)/2}$ with $s < 1$ implies

$$S_1 \rightarrow s^{\bar{\eta}-1} \int_{k,\omega} g|\omega||\psi(k,\omega)|^2. \tag{12}$$

Thus the damping coefficient g flows as

$$\frac{dg}{dl} = (1 - \bar{\eta})g. \tag{13}$$

Since $\bar{\eta} > 1$, this term is clearly irrelevant for g in the vicinity of the unperturbed fixed point. Next consider ΔH_2. Proceeding in the exactly similar manner, one integrates out the electrons and obtains an effective term $g' \int_{k,\omega} \frac{|\omega|}{k} |b(k,\omega)|^2 |b(-k,-\omega)|^2$, where $g' \approx \lambda^2 / E_F$. A simple scaling analysis[21] similar to above yields

$$\frac{dg'}{dl} = (-3 + 2/\nu)g', \tag{14}$$

where ν is the critical exponent associated with scaling of specific heat. Since $\nu > 2/3$ at $O(2)$ critical point,[22] ΔH_2 also turns out to be irrelevant.

Thus we have the remarkable result that a weak coupling to the conduction electrons does not change the critical properties of the transition from that in the insulating magnet. This is in striking contrast to the HMM theory where the Landau damping is strongly relevant. Thus in the present theory, the spin fluctuations are described by a nontrivial interacting fixed point, and hence satisfy scaling (for instance, in ω/T). In contrast, due to the Landau damping, the HMM theory is at the upper critical dimension in $d=2$ (and the fixed point is Gaussian).

The boundaries of the quantum critical region are determined by the requirement that $\{\rho_s/T, \Delta/T, \rho_s/J\} \ll 1$. Here T is the temperature, ρ_s is the spin stiffness corresponding the spinons b (which would be proportional to the spin-exchange coefficient of the original spins $\vec{S}$), and Δ is the gap in the Z_2 spin-liquid phase. Using the scaling relations $\rho_s, \Delta \sim J|r|^\nu$, where $\nu \sim 2/3$ and $r = (1 - t_c/t)$ is a dimensionless measure of the distance from criticality, one obtains $r \ll (T/J)^{3/2} \ll 1$ for being not too far from criticality. Further, though the usual Ginzburg criterion for observing deviations from mean-field behavior applies, even the "mean-field exponents" for the spinons b imply exponents for the physical spin operators $S^+ = b^2$. which are very different than those for $O(2)$ universality class (e.g., $\bar{\eta}_{\text{mean field}} = 1$).
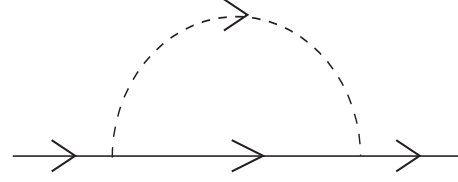

FIG. 3. Lowest-order contribution to the electron's self-energy. The full line is the electron's noninteracting Green's function while the dashed line denotes spin-spin correlator war $\chi(k,\omega)$.

IV. EFFECT OF COUPLING ON CONDUCTION ELECTRONS

Though the coupling to AFM order parameter turns out to be irrelevant for the phase-transition critical properties, it may still affect the electronic properties significantly. First consider the conduction electrons' band structure in the magnetically ordered side. At the mean-field level, the effect of coupling could be captured by the following Hamiltonian:

$$H = \epsilon_k c_k^\dagger c_k + J_K N_0(c_{k+Q}^\dagger \sigma_x c_k + \text{H.c.}). \tag{15}$$

Here N_0 is the AFM order parameter and we have assumed that spins order along $\hat{x}$ direction. The above Hamiltonian is easily diagonalized and one obtains two bands $E^\pm = \frac{\epsilon_k + \epsilon_{k+Q}}{2} \pm \sqrt{(\frac{\epsilon_k - \epsilon_{k+Q}}{2})^2 + J_K^2 N_0^2}$. For any nonzero N_0, the unit cell is doubled and thus one finds electron and hole pockets emerging on the ordered side.[23,24] As N_0 increases, the hole pockets grow in size at the expense of electron pockets, the difference in areas being constant and determined by the density of conduction electrons.[23,24] For small N_0, the separation in momentum space between hole and electron pockets grows in proportion to $N_0 = \langle b^2 \rangle \sim |t - t_c|^{\bar{\beta}}$, where t is the tuning parameter for transition and $\bar{\beta} \approx 0.83$.[20] Thus the separation grows much more slowly compared to a conventional HMM scenario where the AFM order $N_0 \sim |t - t_c|^\beta$ with $\beta \sim 0.35$ corresponding to three-dimensional (3D) X-Y universality class.

Though potentially there are many other interesting questions to be answered on the magnetically ordered side (e.g., the limit $N_0 \rightarrow 0$ and $k, \omega \rightarrow 0$ for various correlation functions may not commute[23]), our focus in the remaining paper would be on the quantum critical regime. We begin by calculating single-particle Green's function for conduction electrons in the quantum critical regime.

A. Single-particle Green's function

Qualitatively, since critical fluctuations are centered around the ordering wave vector Q, the scattering rate at the Fermi points connected by Q will get enhanced and one would expect that the electron-transport properties might show signature of such hot spots. One simple and useful calculation one might do is to calculate the one-particle Green's function $G(k, i\omega_n)$ for a conduction electron perturbatively in the coupling g.

The lowest-order term that contributes to the self-energy $\Sigma(k, i\omega_n) = G_0^{-1}(k, i\omega_n) - G^{-1}(k, i\omega_n)$ (see Fig. 3) is

$$\Sigma(\mathbf{k}, i\omega_n) = TJ_K^2 \sum_{\mathbf{k}', i\Omega_m} \chi_s(\mathbf{k}', i\Omega_m) G_0(\mathbf{k} - \mathbf{k}', i\omega_n - i\Omega_m),$$ (16)

where $\chi_s(\mathbf{k}, \omega) = \langle S^+(\mathbf{k}, \omega) S^-(\mathbf{k}, \omega) \rangle = \langle \psi^*(\mathbf{k} - \mathbf{Q}, \omega) \psi(\mathbf{k} - \mathbf{Q}, \omega) \rangle$ is the dynamic susceptibility of local moments and can be read off from Eq. (7) while $G_0(\mathbf{k}, i\omega_n) = \frac{1}{i\omega_n - \epsilon_k}$ is electron's unperturbed Green's function. Note that here $i\omega_n$ is a fermionic while $i\Omega_m$ is a bosonic Matsubara frequency.

It is useful to go to the spectral representation for $\chi(\mathbf{k}, \Omega)$,

$$\chi(\mathbf{k}, i\Omega_m) = \int \frac{A_b(\mathbf{k}, \omega) d\omega}{\omega - i\Omega_m},$$

where

$$A_b(\mathbf{k}, \omega) = C \frac{\theta(\omega^2 - c^2|\mathbf{k} - \mathbf{Q}|^2)}{(\omega^2 - c^2|\mathbf{k} - \mathbf{Q}|^2)^{1 - \bar{\eta}/2}},$$ (17)

where C is a constant of $O(1)$. Substituting Eq. (17) and expression for $G_0(\mathbf{k}, i\omega_n)$ in Eq. (16), one obtains

$$\Sigma(\mathbf{k}, i\omega_n) \approx J_K^2 \int_{\omega, \mathbf{q}} \frac{[n_b(\omega') + 1 - n_f(\epsilon_{k+q+Q})] \theta(\omega'^2 - q^2)}{[(i\omega_n - \omega' - \epsilon_{k+q+Q})(\omega'^2 - c^2 q^2)^{1 - \bar{\eta}/2}]},$$ (18)

where n_b and n_f are Bose and Fermi functions, respectively, and $\epsilon_k \equiv E_k - \mu$ is the single-particle energy. On physical grounds, it is expected that $\mathrm{Im}\,\Sigma(\mathbf{k}, i\omega_n)$ would be significant only for values of $\mathbf{k}$ such that both $\mathbf{k}$ and $\mathbf{k} + \mathbf{Q}$ lie on the Fermi surface. Such points on the Fermi surface are generally called hot spots. Let $\mathbf{k}_0$ be one such point. We find (Appendix B) that at $T = 0$,

$$\mathrm{Im}\,\Sigma(\mathbf{k}, i\omega_n = \omega^+) \sim J_K^2 \,\mathrm{sign}(\omega) \omega^{\bar{\eta}}$$ (19)

for $|\mathbf{k} - \mathbf{k}_0| \lesssim \omega/c$ while

$$\mathrm{Im}\,\Sigma(\mathbf{k}, i\omega_n = \epsilon_k^+ = 0) \sim J_K^2 T^{\bar{\eta}}$$ (20)

at a nonzero temperature T for $\mathbf{k} \in$, the Fermi Surface and satisfying $|\mathbf{k} - \mathbf{k}_0| \lesssim T/c$. For points $\mathbf{k}$ near the Fermi surface away from the hot spots, the self-energy has the usual Landau Fermi-liquid form, namely, $\mathrm{Im}\,\Sigma(\mathbf{k}, i\omega_n = \omega^+) \sim \omega^2 + T^2$. Evidently, the hot spots acquire a width $O(T)$ at nonzero but small temperature T. Perhaps most importantly, the electronic quasiparticle maintains its integrity at the transition at all points on the Fermi surface since $\frac{\partial \mathrm{Im}\,\Sigma(\omega)}{\partial \omega}\big|_{\text{hot spot}} \sim \omega^{\bar{\eta} - 1} \to 0$ as $\omega \to 0$.

Finally, the real part of self-energy at $T = 0$ for points near a hot spot behaves as $\mathrm{Re}\,\Sigma(\mathbf{k} \approx \mathbf{k}_0, \omega) \sim J_K^2(-\omega \Lambda^{\bar{\eta} - 1} + \omega^{\bar{\eta}})$, where $\Lambda \sim v_F/a$ is an ultraviolet frequency cutoff and a is the lattice spacing (Appendix B).

B. Thermodynamic and transport properties of electronic quasiparticles

1. Specific heat

The correction to the internal energy of the electronic system is given by

$$\Delta U = \sum_{\mathbf{k}} \Delta \epsilon_{\mathbf{k}} n_f(\epsilon_{\mathbf{k}}),$$ (21)

where $\Delta \epsilon_{\mathbf{k}} = \mathrm{Re}\,\Sigma(\mathbf{k}, i\omega_n = \epsilon_k^+)$ is the correction to the single-particle energy levels. As argued above that the regions which contribute primarily to $\Delta \epsilon_{\mathbf{k}}$ are located around hot spots (denoted $\mathbf{k}_0$ here and above) and have a width proportion to temperature T. Setting up a polar coordinate system $q - \phi$ near one such $\mathbf{k}_0$ and noticing that $\epsilon_k \approx v_F q \cos(\phi)$, the expression for ΔU may be written as

$$\Delta U \approx \int_0^{2\pi} d\phi \int_0^T q \, dq \frac{J_K^2 \{\Lambda^{\bar{\eta} - 1} q \cos(\phi) - [q \cos(\phi)]^{\bar{\eta}}\}}{e^{\beta q \cos(\phi)} + 1}$$ (22)

which immediately yields $\Delta U \approx J_K^2(\Lambda^{\bar{\eta} - 1} T^3 - T^{\bar{\eta} + 2})$ just from scaling. Thus the correction to electronic specific heat ΔC_v is

$$\Delta C_v = \frac{\partial U}{\partial T}$$ (23)

$$\approx J_K^2(\Lambda^{\bar{\eta} - 1} T^2 - T^{\bar{\eta} + 1})$$ (24)

which has the same temperature dependence as the contribution from the local moments ($\propto T^2$) at low temperatures while being subdominant to the contribution from the electronic system in the absence of any coupling to spin fluctuations ($\propto T$) at low temperatures.

2. Spin susceptibility

The dominant contribution to the spin susceptibility comes from the critical spin fluctuations χ_s. Here we calculate the subleading contribution χ_e arising from the conduction electrons. The spin susceptibility for a noninteracting Fermi system $\chi_{eo}(\mathbf{k}, \omega)$ at $T = 0$ for $\omega \ll k$ is given by $\chi_{eo}(\mathbf{k}, \omega) \approx -N(E_F)(1 + iC\omega)$, where C is constant. To $O(J_F^2)$, the correction to this result due to coupling to critical fluctuations could be calculated by replacing the noninteracting Green's function by the full interacting one in the "polarization bubble" diagram for the free-electron susceptibility. Thus

$$\chi_e(\mathbf{k}, i\omega_n) = T \sum_{\mathbf{q}, i\nu_r} G(\mathbf{k} + \mathbf{q}, i\omega_n + i\nu_r) G(\mathbf{q}, i\nu_r).$$ (25)

Expressing the Green's function in terms of spectral function $A(\mathbf{k}, \omega)$ and doing the summation over Matsubara frequencies $i\nu_r$,

$$\chi_e(\mathbf{k}, i\omega_n) = \int_q \int_{\Omega_1, \Omega_2} \left[\frac{n_f(\Omega_1) - n_f(\Omega_2)}{i\omega_n + \Omega_2 - \Omega_1} \right] A(\mathbf{k} + \mathbf{q}, \Omega_1) A(\mathbf{q}, \Omega_2).$$ (26)

The spectral function $A(\mathbf{k}, \omega) = \mathrm{Im}\,G(\mathbf{k}, \omega^+)$ is expressed simply in terms of inverse single-particle lifetime $\Gamma(\mathbf{k}, \omega)$ as

$$A(\mathbf{k}, \omega) = \frac{\Gamma(\mathbf{k}, \omega)}{\Gamma^2(\mathbf{k}, \omega) + (\omega - \epsilon_k')^2},$$ (27)

where $\epsilon_k' = \epsilon_k + \mathrm{Re}\,\Sigma(\mathbf{k}, \omega = \epsilon_k^+) \propto \epsilon_k$ to the leading order. The imaginary part of $\chi(\mathbf{k}, \omega)$ is thus obtained as

$$\text{Im } \chi_e(\boldsymbol{k}, \omega) = \int_q \int_\Omega [n_f(\Omega) - n_f(\Omega - \omega)]$$

$$\times A(\boldsymbol{k} + \boldsymbol{q}, \Omega) A(\boldsymbol{q}, \Omega - \omega). \quad (28)$$

Since $A(\boldsymbol{k}, \omega)$ is modified significantly due to critical fluctuations only near hot points, the above integral would get important contribution beyond the free electron result only if $\boldsymbol{k} \approx \boldsymbol{Q}$. Further, the momentum integration over $\boldsymbol{q}$ would get significant contribution only from a thin region around hot spots of linear dimensions ω. Using the fact that for $\epsilon_k \lesssim \omega$ and $\boldsymbol{k}$ belonging to hot spot, $A(\boldsymbol{k}, \omega) \sim 1/\omega^{2-\bar{\eta}}$, one readily obtains $\text{Im } \chi_e(\boldsymbol{k} \approx \boldsymbol{Q}, \omega) \sim \text{Im } \chi_{eo}(\boldsymbol{k} \approx \boldsymbol{Q}, \omega) + \omega^{2\bar{\eta}-1}$. Since $\bar{\eta} > 1$, this implies that to the leading order $\chi_e = \chi_{eo}$ and thus the leading correction to the total spin susceptibility is dominated by the free-electron susceptibility.

3. dc conductivity

Under the relaxation-time approximation for the linearized Boltzmann's transport equation in the presence of an electric field $\boldsymbol{E}$, the conductivity for a Fermi liquid at low temperatures is given by $\sigma = \frac{ne^2 \langle \tau_k \rangle_{FS}}{m}$, where n is the carrier density and $\langle \tau_k \rangle_{FS}$ is the relaxation time τ_k averaged over the Fermi surface. The relaxation time τ is proportional to the inverse scattering rate but in general, one needs to weight the scattering rate by the factor $[1 - \cos(\theta)]$, where θ is the scattering angle. Here we neglect this factor since the scattering due to spin excitations have $\theta \sim \theta_0 = O(1)$, where θ_0 is the angle subtended by the hot spots $\boldsymbol{k_0}$ and $\boldsymbol{k_0} + \boldsymbol{Q}$ at the origin of the Brillouin zone. Further the leading contributions of the other two scattering processes, we consider below, namely, electron-electron and impurity scattering is unchanged by the introduction of this factor.

Near the hot points, the scattering rate $1/\tau$ of electronic quasiparticles due to critical spin fluctuations is proportional to $\text{Im } \Sigma \sim T^{\bar{\eta}}$. Thus $\tau_M/\tau \approx x + (T/\Gamma)^{\bar{\eta}} \equiv x + t^{\bar{\eta}}$, where x measures the scattering rate due to (weak) disorder and is isotropic while τ_M and Γ are characteristic scattering time and energy scale, respectively. In the cold regions, one expects typical Fermi-liquid behavior, thus $\tau_M/\tau \approx x + (T/\Gamma)^2 = x + t^2$. From this and using the fact that the width of the hot spots is proportional to t, one obtains the following expression for the conductivity within the linearized relaxation-time approximation:

$$\sigma \propto \frac{t}{x + t^{\bar{\eta}}} + \frac{1 - t}{x + t^2}. \quad (29)$$

Following Rosch,[25] we consider the two limits, $x < t^2 < 1$ and $t < x < 1$. We find that in both limits, the resistivity $\rho = 1/\sigma \propto x + t^2$. Thus the Fermi-liquid result $\rho \propto T^2$ remains valid at low temperatures. This result is very different compared to that for the resistivity near the $z = 2$ quantum critical point described within Hertz-Millis-Moriya theory where one obtains $\rho \propto x + T^2$ in the limit $x < t^2 < 1$ while it is *anomalous* and $\propto x + T^{3/2}$ in the limit $t < x < 1$.[25]

V. SUMMARY AND DISCUSSION

A crucial ingredient in the example presented in this paper for the magnetic transition in the presence of metallic coupling has been the fact that (1) for the parent insulating system, the spinons are deconfined at the phase transition and that (2) the critical exponent $\bar{\eta} > 1$. We found that due to these facts conduction electrons become dynamically decoupled from the underlying spin system at low energies and therefore the phase transition remains in the exotic universality class XY^* even in the presence of coupling to conduction electrons. Further we found that the metallic coupling favors the ordered phase over paramagnetic spin liquid. We also analyzed the effect of critical fluctuations on the conduction electrons and found that the Landau quasiparticle survives at the phase transition. This enabled us to calculate corrections to various thermodynamic and transport properties of the conduction electrons due to critical fluctuations. Overall, we found that to the leading order the critical fluctuations do not modify any thermodynamic or transport property of the conduction electrons and all the results from standard Fermi-liquid theory remain valid.

Contrasting our example with the conventional HMM (Ref. 1) theory, one immediately notices rather stark differences. The HMM theory describes the onset of spin-density wave order from a paramagnet heavy Fermi liquid phase with a large Fermi surface. Spinon excitations do not exist in either phase. The Landau damping of the spin-density wave fluctuations leads to a dynamic critical exponent $z = 2$. The HMM theory is then right at the upper critical dimension in $d = 2$, and the physics is determined by marginal terms associated with interactions between the order-parameter fluctuations. Nontrivial scaling of the order-parameter fluctuations is not expected within the HMM theory. On the other hand, in our example, the paramagnetic metallic phase has a small Fermi surface and gapped deconfined spinon excitations though the magnetically ordered phase is a conventional phase with no spinon excitations. Nevertheless, the critical theory for the transition is most usefully formulated in terms of the spinon degrees of freedom. The theory has $z = 1$ and is below its upper critical dimension. Hence the order-parameter fluctuations naturally exhibit ω/T scaling for various critical properties.

What would be some of the experimental signatures of the phase transition described in our paper? Because of the large critical exponent $\bar{\eta} \approx 1.45$, one would observe broad magnon linewidths near the transition. We note that a proposal for a similar phase transition was made in the context of the material Cs_2CuCl_4 where one observes long tails in the spin-wave spectrum that extend to very high frequencies.[26,27] Furthermore, since the exponent $\bar{\beta} \approx 0.83$ is also appreciably bigger than that for the $O(2)$ transition in 3D, one would see order parameter falling off very slowly near the phase transition. As pointed out in the Sec. IV B, the electrical transport in the quantum critical regime would show Fermi-liquid behavior in contrast to the materials which obey Hertz-Millis-Moriya theory. Finally, we note that the physics of the phase transition is related to the presence of a Z_2 spin liquid (FL*) on the disordered side of the transition. Hence a direct transition from a magnet to a paramagnet which has hints of being a fractionalized Fermi liquid (FL*) could also be an indication for the transition described in this paper. In this regard, one may note that the FL* phase satisfies a Luttinger

theorem which is different than that for the heavy Fermi liquid.[11]

In summary, we provide an example of a magnetic phase transition in a metallic system where the conventional HMM theory fails and which at the same time shares some of the properties with rather intriguing phases[15] and phase transitions[3–5] in itinerant metallic systems.

APPENDIX A

Here we analyze the phase diagram for action S is Eq. (9) for $g \ll 1$. Rewriting it as

$$S = \sum_{\langle(\mathbf{x},\tau),(\mathbf{x}',\tau')\rangle} -t[b^\dagger(\mathbf{x},\tau)b(\mathbf{x}',\tau') + \text{H.c.}] + g \sum_{\mathbf{x},\tau,\tau'} K(\tau - \tau')$$
$$\times [b(\mathbf{x},\tau) \cdot b^\dagger(\mathbf{x},\tau')][b(\mathbf{x},\tau) \cdot b^\dagger(\mathbf{x},\tau')].$$

The above form is amenable to a large-N generalization which allows us to do a more controlled calculation. Thus we consider N species of bosons b_α with the following action S_N,

$$S_N = \sum_{\langle(\mathbf{x},\tau),(\mathbf{x}',\tau')\rangle\alpha} -t[b_\alpha^\dagger(\mathbf{x},\tau)b_\alpha(\mathbf{x}',\tau') + \text{H.c.}]$$
$$+ \frac{g}{N} \sum_{\mathbf{x},\tau,\tau',\alpha,\beta} K(\tau - \tau')[b_\alpha(\mathbf{x},\tau) \cdot b_\alpha^\dagger(\mathbf{x},\tau')]^2.$$

Thus the $O(2)$ symmetry of the original action has been replaced with $O(2N)$ symmetry. At $N = \infty$, the above action reduces to the simple form,

$$S_N = \sum_{\mathbf{k},\omega,\mathbf{k}',\omega',\alpha} b_\alpha^\dagger(\mathbf{k},\omega)b_\alpha(\mathbf{k},\omega)[\epsilon(\mathbf{k},\omega) + gK(\omega$$
$$+ \omega')G_\alpha(\mathbf{k}',\omega')].$$

Here $K(\omega) = |\omega|$ is the Fourier transform of $K(\tau)$, $\epsilon(\mathbf{k},\omega) = -2t[\cos(k_x) + \cos(k_y) + \cos(\omega)]$, and $G_\alpha(\mathbf{k},\omega) = \langle b_\alpha(\mathbf{k},\omega)^\dagger b_\alpha(\mathbf{k},\omega)\rangle$. Since we are approaching the phase transition from the paramagnetic side, $G_\alpha(\mathbf{k},\omega) \equiv G(\mathbf{k},\omega)$ is independent of α.

First consider S_N at $g = 0$. Imposing the constraint $b^\dagger b = 1$ using a Lagrange multiplier λ, the action becomes,

$$S_0 = \sum_{\mathbf{k},\omega,\alpha} b_\alpha^\dagger(\mathbf{k},\omega)b_\alpha(\mathbf{k},\omega)[\epsilon(\mathbf{k},\omega) + \lambda]. \tag{A1}$$

Imposing self-consistency for $G(\mathbf{k},\omega)$ yields

$$\sum_{\mathbf{k},\omega} \frac{1}{\lambda - 2t[\cos(k_x) + \cos(k_y) + \cos(\omega)]} = 1. \tag{A2}$$

At the phase transition, the gap to excitations closes and hence $\lambda = 6t$. Thus the critical $t = t_{0c}$ at $g = 0$ is given by

$$t_{0c} = \sum_{\mathbf{k},\omega} \frac{1}{6 - 2[\cos(k_x) + \cos(k_y) + \cos(\omega)]}. \tag{A3}$$

Next consider the self-consistency condition at the phase transition for a nonzero but small g. One finds

$$\sum_{\mathbf{k},\omega} \frac{1}{6t_c - 2t_c[\cos(k_x) + \cos(k_y) + \cos(\omega)] + gF(\omega)} = 1, \tag{A4}$$

where $F(\omega) = \sum_{\mathbf{k}',\omega'}\{[K(\omega+\omega') - K(\omega')]G(\mathbf{k}',\omega')\}$ Using the expression for G at $g = 0$, one readily finds that $F(\omega) > 0 \,\forall\, \omega$. Thus to $O(g)$, one reaches the conclusion that $t_c = t_{0c} - \alpha g$, where α is a positive constant. Thus the phase boundary curves as shown in Fig. 1.

APPENDIX B

Using Eq. (18), the imaginary part of the self-energy is

$$\text{Im}\,\Sigma(\mathbf{k}, i\omega_n = \omega^+)$$
$$\approx J_K^2 \int_{\omega',\mathbf{q}} \frac{[n_b(\omega') + 1 - n_f(\epsilon_{\mathbf{k}+\mathbf{q}+\mathbf{Q}})]\theta(\omega'^2 - c^2q^2)}{(\omega'^2 - c^2q^2)^{1-\bar{\eta}/2}}$$
$$\times \delta(\omega - \omega' - \epsilon_{\mathbf{k}+\mathbf{Q}+\mathbf{q}}). \tag{B1}$$

First consider $\text{Im}\,\Sigma(\mathbf{k_0}, i\omega_n = \omega^+)$ at $T = 0$, where $\mathbf{k_0}$ is a hot spot, i.e., both $\mathbf{k_0}$ and $\mathbf{k_0} + \mathbf{Q}$ lie on the Fermi surface. Let us resolve $\mathbf{q}$ in components $q_\parallel$, $q_\perp$ parallel and perpendicular to the Fermi surface, respectively. Since the important contribution at small ω comes from $\mathbf{q}$ located near the Fermi surface, $\epsilon(\mathbf{k_0} + \mathbf{Q} + \mathbf{q}) \approx v_F q_\perp$. Substituting this and the expressions for n_b and n_f in $T \to 0$ limit into the Eq. (B1) one finds,

$$\text{Im}\,\Sigma(\mathbf{k_0}, i\omega_n = \omega^+) \approx J_K^2\,\text{sign}(\omega) \int dq_\parallel dq_\perp$$
$$\times \frac{\theta(\omega - v_F q_\perp)\theta(v_F q_\perp)\theta[(\omega - v_F q_\perp)^2 - c^2 q^2]}{[(\omega - v_F q_\perp)^2 - c^2 q^2]^{1-\bar{\eta}/2}}. \tag{B2}$$

The integration over $q_\parallel$ could be done easily by just rescaling the variables yielding

$$\text{Im}\,\Sigma(\mathbf{k_0}, i\omega_n = \omega^+) \approx J_K^2\,\text{sign}(\omega) \int_0^{\omega/v_F + c} dq_\perp [(\omega - v_F q_\perp)^2$$
$$- c^2 q_\perp^2]^{(\bar{\eta}-1)/2} \approx J_K^2\,\text{sign}(\omega)\omega^{\bar{\eta}}.$$

Similarly consider $\text{Im}\,\Sigma(\mathbf{k}, i\omega_n = \omega^+)$ when $\mathbf{k}$ is located away from hot spots which implies $\epsilon_{\mathbf{k}+\mathbf{Q}} \neq 0$. Following the same procedure as above, one finds that the Heaviside function imposes the condition that for $\text{Im}\,\Sigma$ to be nonzero, $\omega \gtrsim \epsilon_{\mathbf{k}+\mathbf{Q}}$. For $\mathbf{k}$'s that do satisfy this condition, $\text{Im}\,\Sigma$ could be approximated by the same expression as above and is thus $\propto \bar{\omega}^\eta$ for small ω.

Next we consider $\text{Im}\,\Sigma(\mathbf{k_0}, i\omega_n = \epsilon_{\mathbf{k_0}}^+ = 0)$ at finite but small temperatures,

$$\text{Im } \Sigma(\boldsymbol{k_0}, i\omega_n = 0^+) \approx J_K^2 \int_{\mathbf{q}}$$

$$\times \frac{[n_b(-\epsilon_{\boldsymbol{k_0}+\boldsymbol{q}+\boldsymbol{Q}}) + 1 - n_f(\epsilon_{\boldsymbol{k_0}+\boldsymbol{q}+\boldsymbol{Q}})] \theta(\epsilon_{\boldsymbol{k_0}+\boldsymbol{q}+\boldsymbol{Q}}^2 - c^2 q^2)}{(\epsilon_{\boldsymbol{k_0}+\boldsymbol{q}+\boldsymbol{Q}}^2 - c^2 q^2)^{1-\bar{\eta}/2}}.$$

The naive approximation $\epsilon(\boldsymbol{k_0}+\boldsymbol{Q}+\boldsymbol{q}) \approx v_F q_\perp$ yields zero for the above integral because such an approximation leads to a spurious symmetry $q_\perp \leftrightarrow -q_\perp$. It is easy to verify that using the full expression for $\epsilon(\boldsymbol{k_0}+\boldsymbol{Q}+\boldsymbol{q}) = q^2/2m + v_F q_\perp$, the integrand is nonzero as it should be on physical grounds. The scale dependence on T could be easily extracted by rescaling the variables and one finds,

$$\text{Im } \Sigma(\boldsymbol{k_0}, i\omega_n = 0^+) \sim J_K^2 T^{\bar{\eta}}. \tag{B3}$$

At points that are located away from the hot spots, Fermi/Bose functions in the above equation impose that $\text{Im } \Sigma(\mathbf{k_0}, i\omega_n = 0^+) \approx 0$ if $|\boldsymbol{k} - \boldsymbol{k_0}| \gtrsim T/c$.

Finally, the real part of self-energy for points in the vicinity of hot spots at finite temperature could be calculated from the corresponding imaginary part using the Kramers-Kronig relation,

$$\text{Re } \Sigma(\boldsymbol{k} \approx \boldsymbol{k_0}, \omega) = \frac{1}{\pi} P \int_\nu \frac{|\text{Im } \Sigma(\boldsymbol{k_0}, \nu)|}{\omega - \nu}, \tag{B4}$$

where the symbol P denotes the principal value of the integral. Substituting $\text{Im } \Sigma(\boldsymbol{k_0}, \nu) \approx J_K^2 \text{ sign}(\nu) \nu^{\bar{\eta}}$, one obtains $\text{Re } \Sigma(\boldsymbol{k} \approx \boldsymbol{k_0}, \omega) \sim J_K^2(-\omega \Lambda^{\bar{\eta}-1} + \omega^{\bar{\eta}})$, where Λ is an ultraviolet frequency cutoff.

[1] J. A. Hertz, Phys. Rev. B **14**, 1165 (1976); T. Moriya, *Spin Fluctuations in Itinerant Electron Magnetism* (Springer-Verlag, Berlin, 1985); A. J. Millis, Phys. Rev. B **48**, 7183 (1993).

[2] H. v. Löhneysen, A. Rosch, M. Vojta, and P. Wölfle, Rev. Mod. Phys. **79**, 1015 (2007).

[3] O. Stockert, H. v. Löhneysen, A. Rosch, N. Pyka, and M. Loewenhaupt, Phys. Rev. Lett. **80**, 5627 (1998).

[4] A. Schröder, G. Aeppli, E. Bucher, R. Ramazashvili, and P. Coleman, Phys. Rev. Lett. **80**, 5623 (1998).

[5] A. Schröder, G. Aeppli, R. Coldea, M. Adams, O. Stockert, H. v. Löhneysen, E. Bucher, R. Ramazashvili, and P. Coleman, Nature (London) **407**, 351 (2000).

[6] H. v. Löhneysen, T. Pietrus, G. Portisch, H. G. Schlager, A. Schröder, M. Sieck, and T. Trappmann, Phys. Rev. Lett. **72**, 3262 (1994).

[7] H. v. Löhneysen, J. Phys.: Condens. Matter **8**, 9689 (1996).

[8] H. v. Löhneysen, S. Mock, A. Neubert, T. Pietrus, A. Rosch, A. Schröder, O. Stockert, and U. Tutsch, J. Magn. Magn. Mater. **177-181**, 12 (1998).

[9] Q. Si, S. Rabello, K. Ingersent, and J. L. Smith, Nature (London) **413**, 804 (2001); Phys. Rev. B **68**, 115103 (2003).

[10] P. Coleman, C. Pépin, Q. Si, and R. Ramazashvili, J. Phys.: Condens. Matter **13**, R723 (2001).

[11] T. Senthil, S. Sachdev, and M. Vojta, Phys. Rev. Lett. **90**, 216403 (2003).

[12] T. Senthil, M. Vojta, and S. Sachdev, Phys. Rev. B **69**, 035111 (2004).

[13] S. Paschen, T. Lühmann, S. Wirth, P. Gegenwart, O. Trovarelli, C. Geibel, F. Steglich, P. Coleman, and Q. Si, Nature (London) **432**, 881 (2004).

[14] H. Shishido, R. Settai, H. Harima, and Y. Ōnuki, J. Phys. Soc. Jpn. **74**, 1103 (2005).

[15] S. Friedemann, T. Westerkamp, M. Brando, N. Oeschler, S. Wirth, P. Gegenwart, C. Krellner, C. Geibel, and F. Steglich, Nat. Phys. **5**, 465 (2009).

[16] S. Burdin, D. R. Grempel, and A. Georges, Phys. Rev. B **66**, 045111 (2002).

[17] T. Senthil, S. Sachdev, and M. Vojta, Physica B **359–361**, 9 (2005).

[18] P. Coleman, J. B. Marston, and A. J. Schofield, Phys. Rev. B **72**, 245111 (2005); I. Paul, C. Pépin, and M. R. Norman, Phys. Rev. Lett. **98**, 026402 (2007).

[19] T. Senthil and O. Motrunich, Phys. Rev. B **66**, 205104 (2002).

[20] Y. Yamazaki, Phys. Lett. **49A**, 215 (1974); A. Aharony, R. J. Birgeneau, J. D. Brock, and J. D. Litster, Phys. Rev. Lett. **57**, 1012 (1986); Pasquale Calabrese, Andrea Pelissetto, and Ettore Vicari, Phys. Rev. E **65**, 046115 (2002).

[21] S. Sachdev and T. Morinari, Phys. Rev. B **66**, 235117 (2002).

[22] M. Campostrini, M. Hasenbusch, A. Pelissetto, P. Rossi, and E. Vicari, Phys. Rev. B **63**, 214503 (2001).

[23] S. Sachdev, A. V. Chubukov, and A. Sokol, Phys. Rev. B **51**, 14874 (1995).

[24] A. V. Chubukov and D. K. Morr, Phys. Rep. **288**, 355 (1997).

[25] A. Rosch, Phys. Rev. Lett. **82**, 4280 (1999).

[26] R. Coldea, D. A. Tennant, A. M. Tsvelik, and Z. Tylczynski, Phys. Rev. Lett. **86**, 1335 (2001); R. Coldea, D. A. Tennant, and Z. Tylczynski, Phys. Rev. B **68**, 134424 (2003).

[27] S. V. Isakov, T. Senthil, and Y. B. Kim, Phys. Rev. B **72**, 174417 (2005).